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### APPROXIMATE NUMERICAL METHOD TO EVALUATE PIPELINES IN SITES WITH GEOTECHNICAL INSTABILITY PROBLEMS

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#### SUMMARY

*In Colombia there are hundreds of landslides that directly affect hydrocarbon pipelines. The effects caused to this type of structures by ground instability processes depend on the magnitude of the applied loads and the yield stress of the pipe material. A pipeline subjected to lateral loads perpendicular to its length tends to move in the direction of ground movement, thus the failure mode is directly related to the axial tension and bending moment experienced by the pipe. To analyze this problem the simplest method used to estimate the forces developed in the pipe, consists of assuming the duct as a simply supported beam with a uniformly distributed load acting on it, that load causes a deflection in the center of the beam. An approximate numerical procedure is proposed based on the flexibility method used in structural engineering and the pipe is analyzed as a statically indeterminate beam. The methodology presented allows to calculate the displacements that can occur in a pipe immersed in a landslide and from these them estimate the maximum bending moment that develops and compare it with the yield moment. An approximate numerical procedure is proposed based on the flexibility method used in structural engineering and the pipe is analyzed as a statically indeterminate beam.*

Keywords: Bending moment, pipe yield moment.

#### NOMENCLATURE

$a$  Distance between nodes  
 $E$  Modulus of elasticity  
 $I$  Moment of inertia  
 $L$  Beam length  
 $M$  Bending Moment  
 $M_{\max}$  Maximum induced bending moment

$M_y$  Yield moment  
 $W$  Load  
 $i$  Node  
 $q$  Spring reaction  
 $\theta$  Maximum slope  
 $\Delta, y$  Deflection

#### 1. INTRODUCTION

Given the topographic, climatological, geological and tectonic conditions in Colombia, hundreds of landslides occur each year that directly affect hydrocarbon pipelines. The effects caused by instability processes on oil pipelines, polyducts and gas pipelines depend directly on the magnitude of the applied loads and the pipe's material yield stress. Under these circumstances, pipeline failure can be achieved by different mechanisms; one of them due to the increase in orthogonal lateral loads over the length of the pipe caused by the mass of soil or rock mobilized, another common failure mechanism consists of the loss of vertical support caused by the slide, in many cases, pipes can fail due to a combination of these two mechanisms.

A pipeline subjected to lateral loads applied perpendicular to its length tends to move in the direction of ground movement, in response to the applied external loads. Elongation and bending deformations can occur in the pipeline. In such situation, the mode of failure is directly related to the axial stress and bending moment experienced by the pipe. To analyze this problem, currently, the simplest method used to estimate the stresses to which the pipe is subjected consists of assuming the pipeline as an element of constant stiffness, represented by a beam simply supported at its ends on which a uniformly distributed load acts, causing a maximum deflection in the center of the beam.

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To solve this problem, an approximate numerical procedure is proposed based on the flexibility method used in structural engineering, in which joints are included on the structure to make it statically determined and eliminate the slopes in those articulated points. The method allows to calculate a value of the bending moment that best approximates the real situation under study. The aim is to compare the pipe's yield moment with that calculated by applying the proposed flexibility method, then calculate the displacements of failure in 4 different types of pipes.

### 2. Description of the problem and the proposed solution

In many sectors of the country, the pipelines run through places where instability processes occur. A common case of pipeline failures is associated with landslides located in the vicinity of pipelines rights of way caused by land deconfinement on slopes adjacent to streams. Although the pipes are always installed outside the influence of the natural scour processes of the channels, in many cases the changing conditions of the terrain, such as changes in land use and the incidence of triggering factors, lead to the occurrence of landslides that affect the right of way and cause deformations and even the pipes failure.

When it is evident that a pipe is being deformed as a result of ground forces, a mitigation alternative consists of unearthing it in order to facilitate its rearrangement, if it is considered that the pipe is under the action of forces that endanger its integrity, then the tube is cut, and sections added to restore its continuity.

In a process of instability, the pressure of the slipped soil produces an initial deflection of the tube, in good mechanical conditions and if the lateral pressure does not induce torque, the tube offers resistance and opposes the movement of the ground, behaving as an obstacle to the flow of the soil, under this scenario, in the most critical situation, the soil surrounding the pipe can reach the state of failure due to the punching exerted by the tube longitudinally, thus the pressure acting on the pipe corresponds to a passive earth pressure, oriented in the direction of sliding and applied over the longitudinal area of the pipe.

Under conditions of integrity and continuity of its elements, a steel pipe is capable of resisting the applied external forces and in response deforms in the direction of the movement of the ground. As seen in the FIGURE1, where an 8" diameter pipe moved approximately 8 meters in response to the lateral pressure caused by an instability process that impacted the pipe over a

length of about 10 m (width of the avalanche), which removed it from the site in a length of around 40 m without affecting its integrity.



FIGURE1 DISPLACEMENT OF A PIPE DUE TO THE EFFECT OF THE AVALANCHE

To analyze a situation like the one that occurred in the pipeline showed in Figure 1, the geotechnical analysis must determine the magnitude of the stresses induced on the pipe from the evidence of the movement of the axis of the pipe with respect to its initial position, the magnitude of the applied loads, conditions and mechanical properties of the tube material.

The magnitude of the earth passive pressure acting on the pipe can be estimated from the soil properties, the dimensions of the pipe involved and its depth of cover. The bending resisting moment ( $M_u$ ) is calculated based on the resistance of the tube material, its moment of inertia and the diameter. To estimate the maximum bending moment that develops in the section, the common practice is to consider the pipe as a beam of length  $L$ , simply supported and on which a distributed load acts ( $W$ ), with this simplification the maximum moment is calculated as follows:

$$M_{m\acute{a}x} = \frac{WL^2}{8} \quad (1)$$

The calculation of the maximum moment using equation 1, supposes a simplified analysis of the pipe since it facilitates the analysis considering it as a statically determined structure,

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however it results in values that overestimate the maximum moment which must be compared with the bending resistant moment. ( $M_y$ ) and find out the cause of the pipe breakage if this occurs. As seen in Figure 1, the tube is not simply supported at the ends of the pushed section.

### 2.1 Pipe as a statically indeterminate structure

In structural analysis, a statically indeterminate structure is one for which the number of unknown reactions or internal forces is greater than the number of equilibrium equations available for analysis. For statically indeterminate structures the additional equations required to find the unknown reactions are obtained by the relationship between the applied loads and the reactions, with the displacement or rotation at different points on the structure, these additional equations are known as compatibility equations, and they are written in terms of the geometric and physical properties of the structure.

To represent any structure, it is possible to establish an analysis model composed of nodes. The proposed model must satisfy the conditions of equilibrium and compatibility of movements in the nodes. In statically indeterminate structures, the compatibility conditions can only be related to the loads if the moduli of elasticity of the material and the size and shape of the elements that make up the structure are known. In the case of the pipe, it is considered as a beam with a straight circular cylindrical section, characterized by its bending rigidity  $EI$ , supported on numerous springs that represent the surrounding soil. Over the pipe a distributed load of variable magnitude and polygonal shape acts which models the passive pressure acting from a landslide.

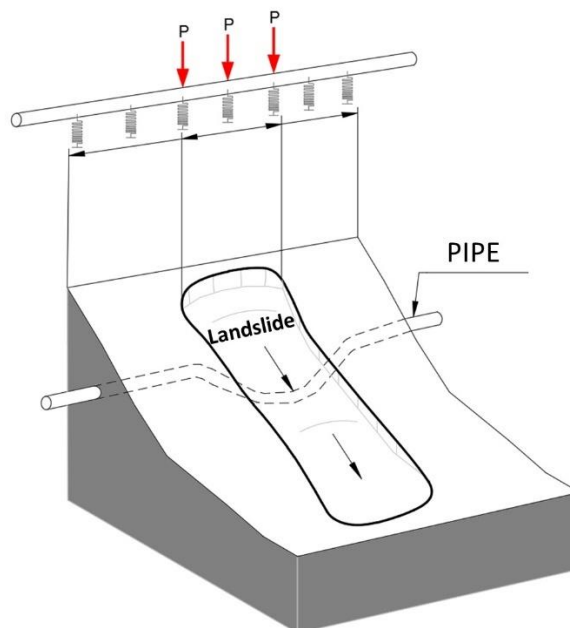


FIGURE2 ANALYSIS PROBLEM IDEALIZATION

In the approach to the analysis of the pipeline as shown in FIGURE2, the number of supports, and consequently of reactions, exceeds the number of equilibrium equations available for analysis, therefore it is configured as a statically indeterminate structure.

The use of statically indeterminate structures in the analysis has the advantage that for a given load, the calculated maximum stress and deflection are generally smaller than in their statically determined counterpart, furthermore indeterminate structures tend to redistribute the load towards the redundant supports representing thus more approximately the real situation. To solve structures of this type, there are various methodologies, one of them known as the flexibility method.

The analysis of statically indeterminate structures by the flexibility method consists of solving equations that satisfy the compatibility and force-displacement requirements in the structure, that equations contain the redundant forces as unknowns. The coefficients of the unknowns are known as flexibility coefficients.

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### 2.2 Calculation methodology

It is proposed to model the interaction mechanism between a pipe and the soil that surrounds it, in a process of instability, considering the pipe as a statically indeterminate beam of length  $L$ . As shown in FIGURE 3, the beam is composed of a number of 20 elements limited by nodes at their ends. Each element has a length  $a$ . A load that simulates the passive earth pressure of the slide acts on the nodes of the elements located in the center of the beam and each node is supported by a spring that simulates the surrounding soil which deforms and reacts to the applied external load.

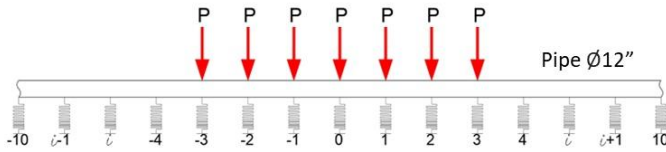


FIGURE 3 IDEALIZATION OF THE PIPE AS A STATICALLY INDETERMINATE BEAM

The moment equilibrium on the beam formed by the nodes  $(i - 1, i, i + 1)$  is calculated to find the average reaction of the spring at the node  $i$ , obtaining:

$$q_i = \frac{a}{8}(q_{i-1} + 6q_i + q_{i+1}) \quad (2)$$

From an elementary beam made up of the nodes  $(i - 1, i, i + 1)$  which is assumed to be fully restrained, artificial joints are included in the node  $i$  and the maximum slopes that occur are found, for this the constitutive equation of the beam is used:

$$EI \frac{d^2y}{dx^2} = M(x) \quad (3)$$

Where  $y$  represents the deflection of the beam and  $M$  is the bending moment of the beam oriented in  $x$  direction. From the moment equilibrium and by double integration of the beam equation, the maximum slopes expression in the supports are obtained:

$$\theta_{i-1} = \frac{dy}{dx} = \frac{\Delta_{i-1} - \Delta_i}{a} = \frac{a}{6EI}(M_{i-1} + 2M_i) \quad (4)$$

$$\theta_{i+1} = \frac{\Delta_{i+1} - \Delta_i}{a} = \frac{dy}{dx} = \frac{a}{6EI}(2M_i + M_{i+1}) \quad (5)$$

Where the moments  $M_{i-1}, M_{i+1}$  are the bending moments that restore the maximum slopes assumed  $\theta_{i-1}, \theta_{i+1}$  respectively.

The angular equilibrium to restore is then  $\theta_{i-1} + \theta_{i+1}$ , therefore adding the previous equations we have:

$$\frac{6EI}{a^2}(\Delta_{i-1} - 2\Delta_i + \Delta_{i+1}) = M_{i-1} + 4M_i + M_{i+1} \quad (6)$$

According to Winkler's spring concept, the soil can be represented as a system of linear springs, under the assumption that contact pressures are proportional to deformations, written in mathematical form:  $\Delta_i = \frac{q_i}{k_i}$ , replacing the above and organizing terms, the angular compatibility expression for a node, written in matrix form, is obtained:

$$\left\{ -g_i \frac{k_i}{k_{i-1}}, 1, 2g_i, 4, -g_i \frac{k_i}{k_{i+1}}, 1 \right\} \begin{Bmatrix} q_{i-1} \\ M_{i-1} \\ q_i \\ M_i \\ q_{i+1} \\ M_{i+1} \end{Bmatrix} = \{0\} \quad (7)$$

Where  $g_i$  is defined as a simplification operator:

$$g_i = \frac{6EI}{a^2 k_i} \quad (8)$$

From the equilibrium of the loaded element made up of the nodes  $(i - 1, i, i + 1)$  it is obtained:

$$M_{i-1} - 2M_i + M_{i+1} = a(P_i - Q_i) \quad (9)$$

Where  $P$  and  $Q$  are load and reaction respectively on the  $i$  spring. Using the equivalence between nodal forces and stresses described in equation 2, the nodal equilibrium equation is obtained, which is written in a matrix form as follows:

$$\left\{ \frac{a^2}{8}, 1, \frac{3a^2}{4}, -2, \frac{a^2}{8}, 1 \right\} \begin{Bmatrix} q_{i-1} \\ M_{i-1} \\ q_i \\ M_i \\ q_{i+1} \\ M_{i+1} \end{Bmatrix} = \{aP_i\} \quad (10)$$

From repeating the analysis for a later node  $i + 1$ , the equations of angular compatibility (Equation 11) and equilibrium (Equation 12) result:

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$$\left\{ -g_{i+1} \frac{k_{i+1}}{k_i}, 1, 2g_{i+1}, 4, -g_{i+1} \frac{k_{i+1}}{k_{i+2}}, 1 \right\} \begin{Bmatrix} q_i \\ M_i \\ q_{i+1} \\ M_{i+1} \\ q_{i+2} \\ M_{i+2} \end{Bmatrix} = \{0\} \quad (11)$$

$$\left\{ \frac{a^2}{8}, 1, \frac{3a^2}{4}, -2, \frac{a^2}{8}, 1 \right\} \begin{Bmatrix} q_i \\ M_i \\ q_{i+1} \\ M_{i+1} \\ q_{i+2} \\ M_{i+2} \end{Bmatrix} = \{aP_i\} \quad (12)$$

The angular compatibility and force balance equations are successively written for all the nodes on which a load  $P$  acts. At nodes where there is no external load applied, the right side of equation 12 is equal to zero.

The boundary conditions of the beam are determined by the constraints applied to the initial and final terminal nodes; in the case of this example the nodes are numbered from  $i = 0$  to  $i = \pm 10$ . Then for the terminal nodes we have:

Equilibrium equation for initial node:

$$q_{i-10} = \frac{a}{8}(3q_{i-10} + q_{i-9}) \quad (13)$$

In a matrix manner it is written as:

$$\left\{ \frac{3a^2}{8}, \frac{a^2}{8}, 1 \right\} \begin{Bmatrix} q_{i-10} \\ q_{i-9} \\ M_{i-9} \end{Bmatrix} = \{0\} \quad (14)$$

Equilibrium equation for final node:

$$q_{i+10} = \frac{a}{8}(q_{i+9} + 3q_{i+10}) \quad (15)$$

In a matrix manner it is written as:

$$\left\{ \frac{a^2}{8}, 1, \frac{3a^2}{8} \right\} \begin{Bmatrix} q_{i+9} \\ M_{i+9} \\ q_{i+10} \end{Bmatrix} = \{0\} \quad (16)$$

With the equations of the initial, final and intermediate nodes, the matrix of flexibility coefficients of the beam is assembled, the value of the bending moment of restitution of the maximum slope and the reaction in the spring is calculated for each node. The method allows estimating the deflection of the beam at each

node, based on the reaction and the rigidity modulus in each spring. In the case of the example, the flexibility coefficient matrix has a size of 40X40, which is solved with the help of a spreadsheet.

### 2.3 Model Description

For the implementation of the approximate numerical model, the behavior under sliding loads of a 12" diameter API 5LX pipe was simulated; the geometric characteristics and elastic properties of the pipe are presented in Table 1.

Pipeline	External diameter (m)	Thickness (m)	Moment of Inertia (m <sup>4</sup> )	Modulus of elasticity (MPa)
12"	0.305	0.0095	0.0004	203395.34

TABLE 1: PIPE CHARACTERISTICS

The loads applied in the model simulate two landslides, one 7m and the other 16m wide. A load magnitude was imposed for which the pipe moves a certain value.

For the case of a 7m wide landslide, maximum displacements in the pipes of 3m, 4m, 12m and 18m were analyzed. For the 16 m wide landslide, the displacements analyzed were 4m, 10m, 18m and 25m.

For each of the displacements, the maximum bending moment that develops in the pipes was obtained and this value was compared with the yield bending moment for pipes API 5LX 52, API 5LX 56, API 5LX 60 and API 5LX 65, the strength characteristics of these pipes were obtained from the book *Pipeline Rules of Thumb Handbook* [3] and are organized in Table 2.

Pipeline type	Specified Min. yield strength (MPa)	Specified Min. yield strength (PSI)
API 5LX 52	358	52000
API 5LX 56	386	56000
API 5LX 60	413	60000
API 5LX 65	448	65000

TABLE 2: RESISTANCE CHARACTERISTICS OF PIPES

The springs that model the soil were characterized with a variable reaction modulus; the adopted values are organized in Table 3.

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Node	0	±1	±2	±3	±4	±5	±6	±7	±8	±9	±10
K (kPa)	10	10	10	10	10	500	1000	2000	4000	5000	10000

TABLE 3: SPRING REACTION MODULES

### 3. RESULTS AND DISCUSSION

Figure 3 and Figure 4 show the displacements of each node obtained for different load magnitudes that simulate landslides of 7m and 16m width.

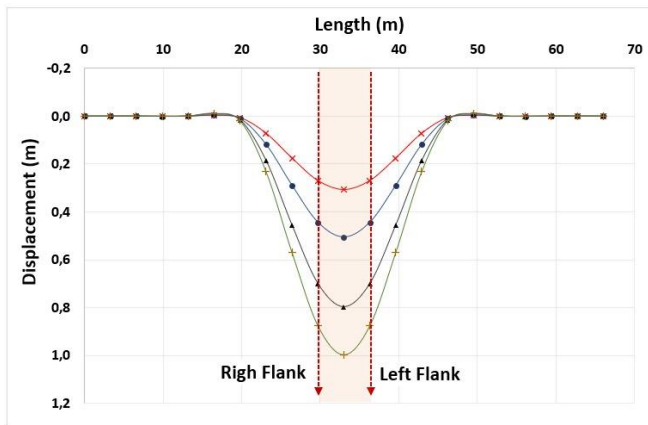


FIGURE 3: DISPLACEMENTS OF A PIPE FOR DIFFERENT LOAD MAGNITUDES, WHICH SIMULATE A 7M WIDTH LANDSLIDE

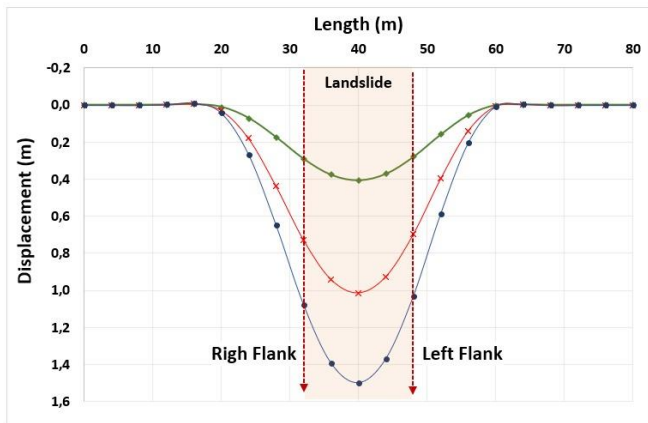


FIGURE 4: DISPLACEMENTS OF A PIPE FOR DIFFERENT LOAD MAGNITUDES, WHICH SIMULATE A 16M WIDTH LANDSLIDE

It is observed in Figure 3 and Figure 4 that in the nodes adjacent to the landslide the displacement is different from zero and decreases as it moves away from the area of the instability process.

For each of the displacements, the maximum bending moment that develops in the pipe was obtained. Figure 5 shows the values of the bending moment at each node obtained for the 12" diameter pipe for a 7m landslide and Figure 6 shows the same graph for a 16m landslide. The maximum bending moment ( $M_{m\acute{a}x}$ ) is located in the center of the landslide, the site where the largest displacements occur.

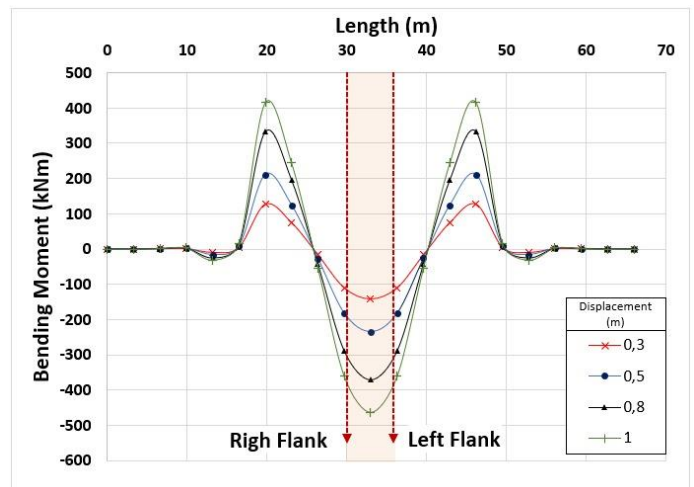


FIGURE 5: BENDING MOMENT OBTAINED FOR A PIPE IN 7M WIDTH LANDSLIDE

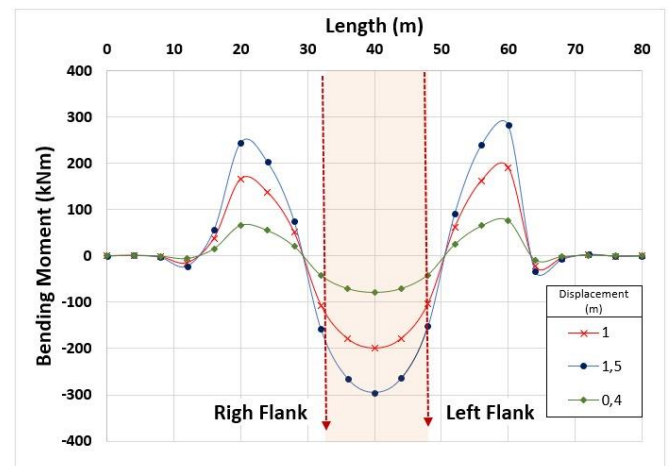
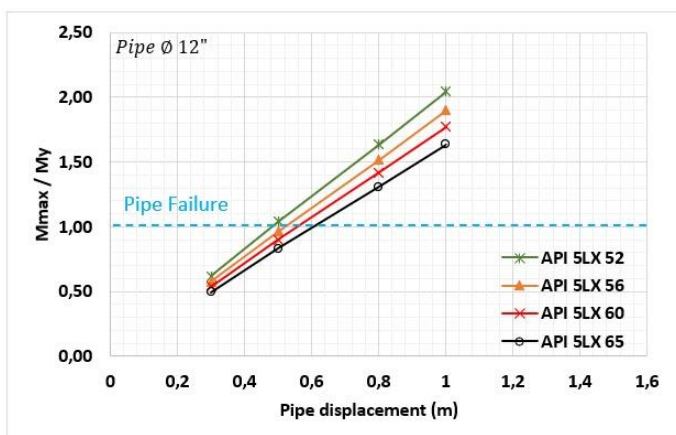


FIGURE 6: BENDING MOMENT OBTAINED FOR A PIPE IN 16M WIDTH LANDSLIDE

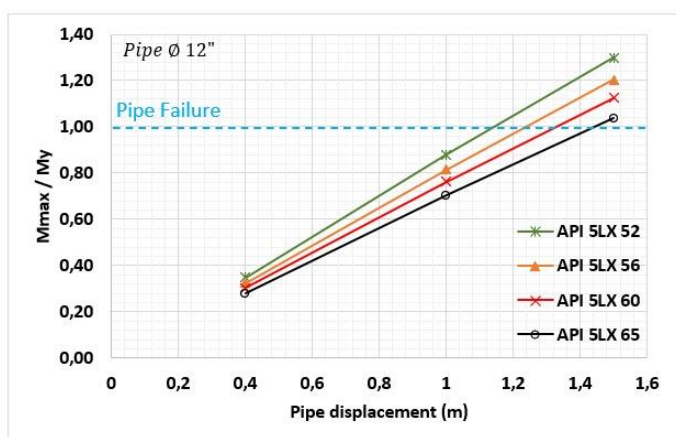
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In Figure 5 and Figure 6 it is observed that the points on the beam where the shear force is maximum, or in other words points where the bending moment is zero, occur in sectors outside the width of the slide.

The values of maximum induced bending moment ( $M_{m\acute{a}x}$ ) obtained by the proposed method were normalized with the value of the yield moment for each pipe ( $M_y$ ). Figure 7 and Figure 8 plot the values of the quotient ( $M_{m\acute{a}x}/M_y$ ) against the maximum displacements for the 4 types of pipes analyzed.



**FIGURE 7:** RATIO ( $M_{m\acute{a}x}/M_y$ ) AGAINST MAXIMUM DISPLACEMENTS, 12" DIAMETER PIPE AND 7M WIDTH LANDSLIDE



**FIGURE 8:** RATIO ( $M_{m\acute{a}x}/M_y$ ) AGAINST MAXIMUM DISPLACEMENTS, 12" DIAMETER PIPE AND 16M WIDTH LANDSLIDE

As seen in Figure 7 for a 12" diameter API 5LX 52 type pipe that reaches a displacement of 0.5m, the maximum induced bending moment ( $M_{m\acute{a}x}$ ) that develops in the pipe reaches the value of the yield moment ( $M_y$ ) which can be interpreted as the pipe failure.

According to the results obtained, in the case of an API 5LX 65 pipe, failure would occur for a displacement of 0.6m.

For the case of a 16m wide landslide, the API 5LX 52 pipe will reach failure when a displacement of 1.1m occurs and for the API 5LX 65 pipe it is 1.5m, as can be read in Figure 8.

### 4. CONCLUSIONS

The use of statically indeterminate structures in the analysis has the advantage that for a given load, the maximum calculated stress and deflection are generally smaller than in their statically determined counterpart. Indeterminate structures tend to redistribute the load towards the redundant supports, thus representing the real situation more approximately.

The methodology presented allows calculating the maximum displacements that can occur in a pipe immersed in a landslide and, from these, estimating the maximum bending moment that develops in the pipe to compare it with the breaking moment. In the calculations, typical soil reaction modulus values for soft soils were adopted and landslide widths of 7m and 16m were studied. Therefore, the results presented are valid specifically for the cases analyzed as examples. Applying the methodology presented to a real case study involves determining the terrain conditions in detail, adequately estimating the soil reaction modules and the geometry and magnitude of the loads imposed by the landslide.

In the analyzes it is assumed that the pipe behaves as a continuous beam, without changes in stiffness along its length.

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- [2] Hibbeler RC Structural Analysis, 3rd Ed.
- [3] McAllister, E.W. Pipeline Rules of Thumb Handbook, 2002